1 Within certain limits, the bob of a simple pendulum of length l may be considered to move with simple harmonic motion of period T, where

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(a) State **one** limitation that applies to the pendulum when this equation is used.

____

____

(1)

(b) Describe an experiment to determine the value of the Earth's gravitational field strength g using a simple pendulum and any other appropriate apparatus.

In your answer you should:

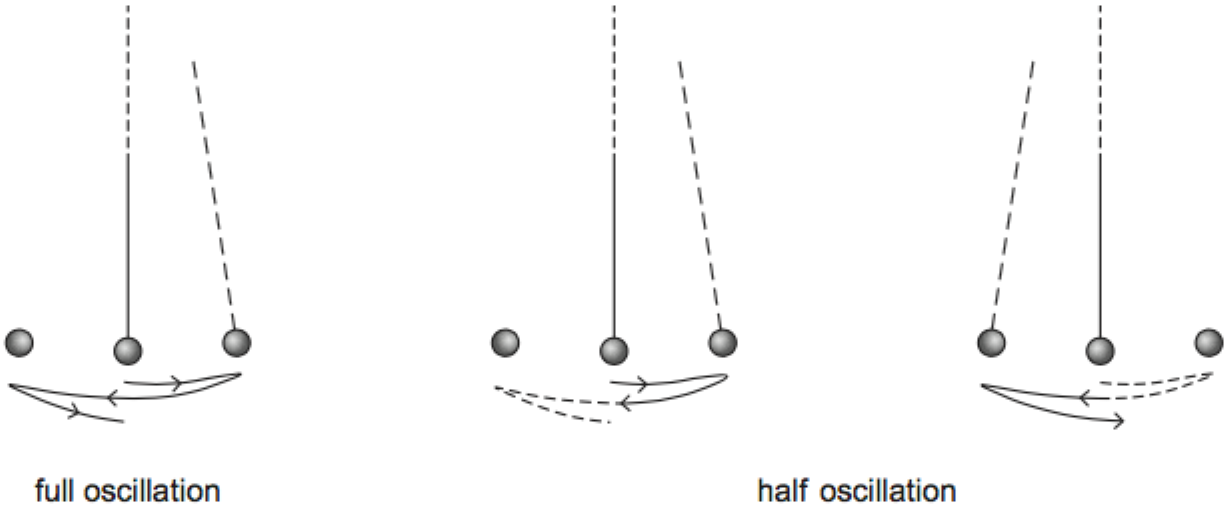
- describe how you would arrange the apparatus
- indicate how you would make the measurements
- explain how you would calculate the value of g by a graphical method
- state the experimental procedures you would use to ensure that your result was accurate.

You may draw a diagram to help you with your answer.

The quality of your written communication will be assessed in your answer.

(6)

(c) When carrying out the experiment in part **(b)**, a student measures the time period incorrectly. Mistakenly, the student thinks that the time period is the time taken for half of an oscillation instead of a full oscillation, as illustrated in the diagram.



Deduce the effect this will have on the value of g obtained from the experiment, explaining how you arrive at your answer.

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(3)

(Total 10 marks)

2 (a) Describe the energy changes that take place as the bob of a simple pendulum makes one complete oscillation, starting at its maximum displacement.

____

____

____

____

(2)

(b)

Figure 1

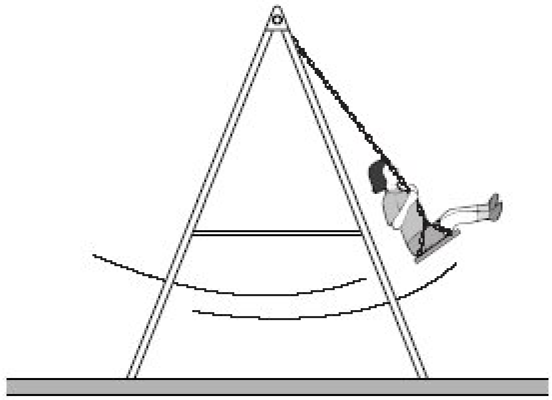


Figure 1 shows a young girl swinging on a garden swing. You may assume that the swing behaves as a simple pendulum. Ignore the mass of chains supporting the seat throughout this question, and assume that the effect of air resistance is negligible.
15 complete oscillations of the swing took 42s.

(i) Calculate the distance from the top of the chains to the centre of mass of the girl and seat. Express your answer to an appropriate number of significant figures.

answer = ____________________ m

(4)

(ii) To set her swinging, the girl and seat were displaced from equilibrium and released from rest. This initial displacement of the girl raised the centre of mass of the girl and seat 250 mm above its lowest position. If the mass of the girl was 18 kg, what was her kinetic energy as she first passed through this lowest point?

answer = ____________________ J

(2)

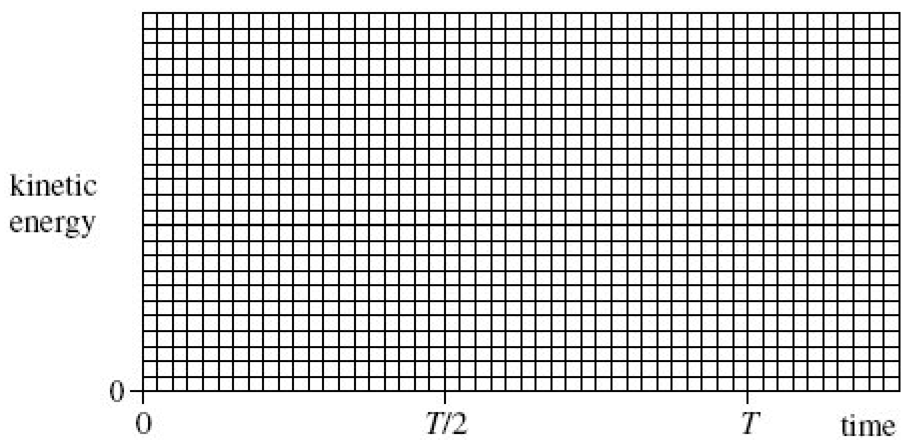
(iii) Calculate the maximum speed of the girl during the first oscillation.

answer = ____________________ m s^{-1}

(1)

(c)

Figure 2



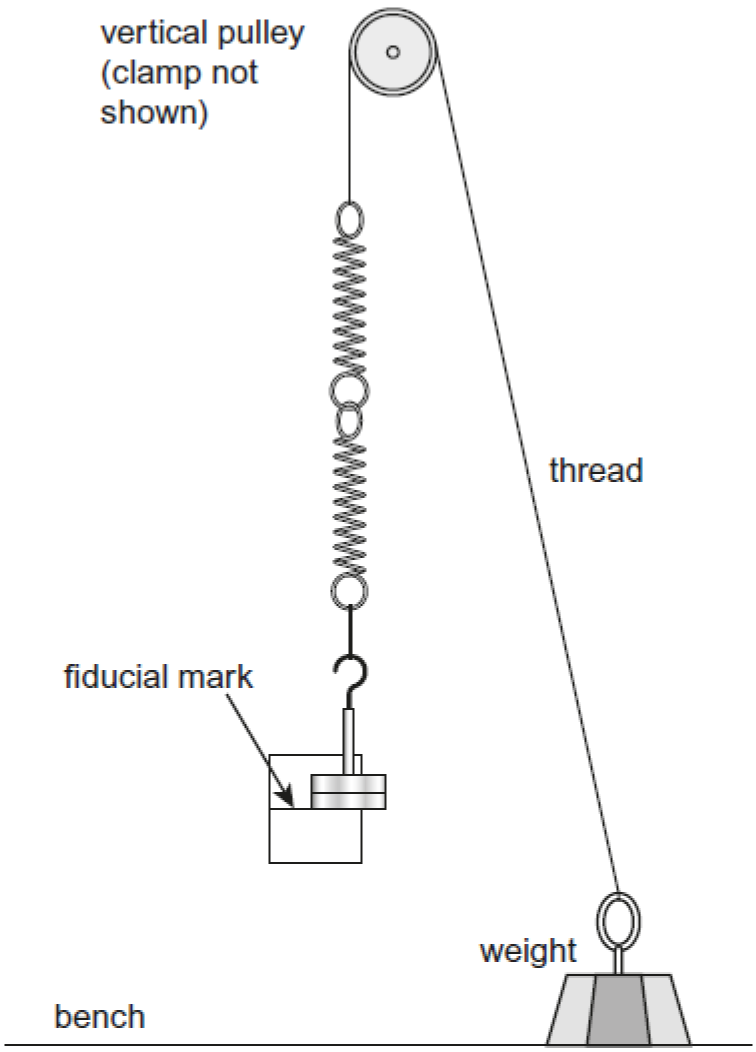
On **Figure 2** draw a graph to show how the kinetic energy of the girl varied with time during the first complete oscillation, starting at the time of her release from maximum displacement. On the horizontal axis of the graph, *T* represents the period of the swing. You do not need to show any values on the vertical axis.

(3)

(Total 12 marks)

3 A student investigates the vertical oscillations of the mass–spring system shown in **Figure 1**.

Figure 1



The system is suspended from one end of a thread passing over a pulley.

The other end of the thread is tied to a weight.

The system is shown in **Figure 1** with the mass at the equilibrium position.

The spring constant (stiffness) is the same for each spring.

(a) Explain why the position of the fiducial mark shown in **Figure 1** is suitable for this experiment.

____

____

____

____

(1)

The table below shows the measurements recorded by the student.

Time for 20 oscillations of the mass-spring system/s				
22.9	22.3	22.8	22.9	22.6

(b) (i) Determine the percentage uncertainty in these data.

percentage uncertainty = ____

(3)

(ii) Determine the natural frequency of the mass-spring system.

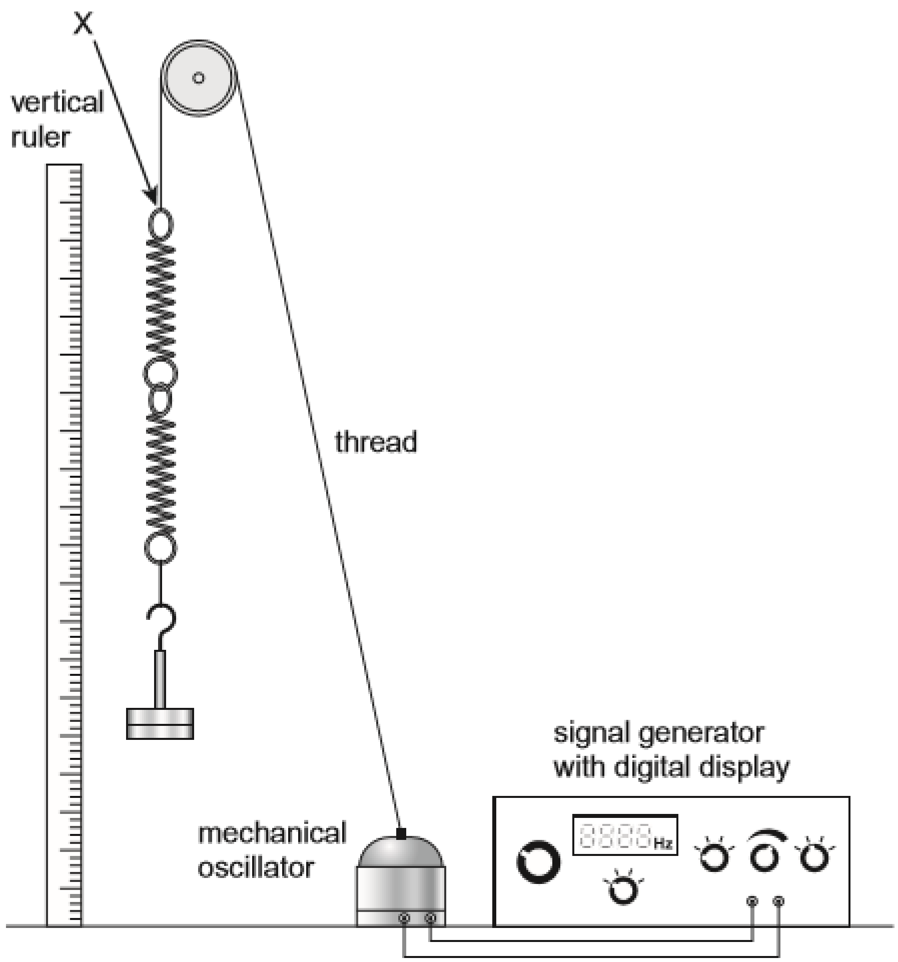
natural frequency = ____

(1)

(c) The student connects the thread to a mechanical oscillator. The oscillator is set in motion using a signal generator and this causes the mass–spring system to undergo forced oscillations.

A vertical ruler is set up alongside the mass–spring system as shown in **Figure 2**. The student measures values of A, the amplitude of the oscillations of the mass as f, the frequency of the forcing oscillations, is varied.

Figure 2



A graph for the student's experiment is shown in **Figure 3**.

(i) Add a suitable scale to the frequency axis.
You should refer to your answer in part **(b)(ii)** and note that the scale starts at 0 Hz.

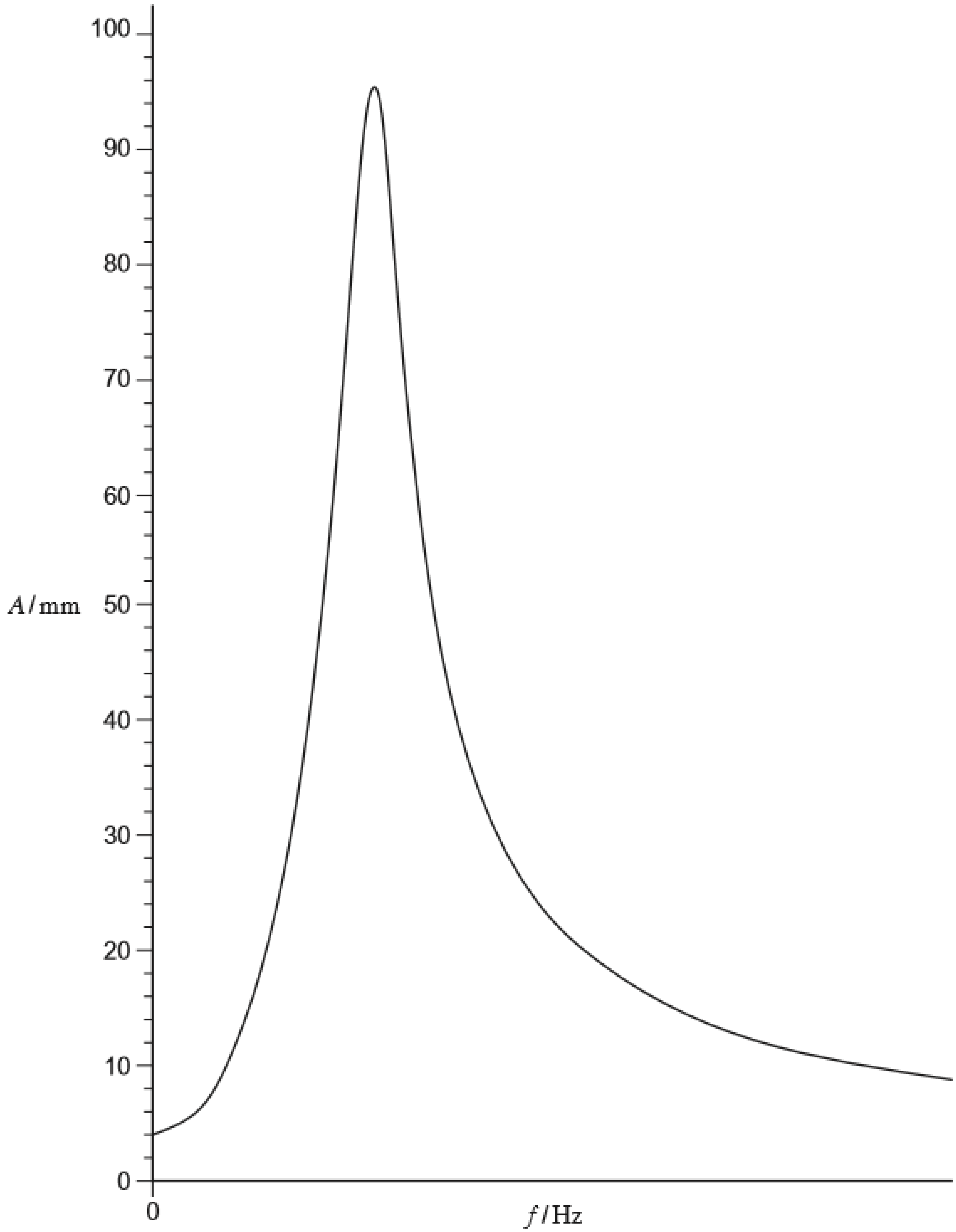
(1)

(ii) Deduce from **Figure 3** the amplitude of the oscillations of X, the point where the mass–spring system is joined to the thread.
You should assume that the length of the thread is constant.

amplitude of X = ____

(1)

Figure 3



(d) (i) State and explain how the student was able to determine the accurate shape of the graph in the region where *A* is a maximum.

___

___

___

___

(2)

(ii) The student removes one of the springs and then repeats the experiment.

Add a new line to **Figure 3** to show the graph the student obtains.

You may wish to use the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(2)

(Total 11 marks)

4 To celebrate the Millennium in the year 2000, a footbridge was constructed across the River Thames in London. After the bridge was opened to the public it was discovered that the structure could easily be set into oscillation when large numbers of pedestrians were walking across it.

(a) What name is given to this kind of physical phenomenon, when caused by a periodic driving force?

___

(1)

(b) Under what condition would this phenomenon become particularly hazardous? Explain your answer.

___

___

___

___

___

___

(4)

(c) Suggest **two** measures which engineers might adopt in order to reduce the size of the oscillations of a bridge

measure 1 ____

____

measure 2 ____

____

(2)
(Total 7 marks)

5 (a) A vibrating system which is experiencing forced vibrations may show *resonance*.

Explain what is meant by

forced vibrations ____

____

resonance ____

____

____

____

(3)

(b) (i) Explain what is meant by *damping*.

____

____

(ii) What effect does damping have on resonance?

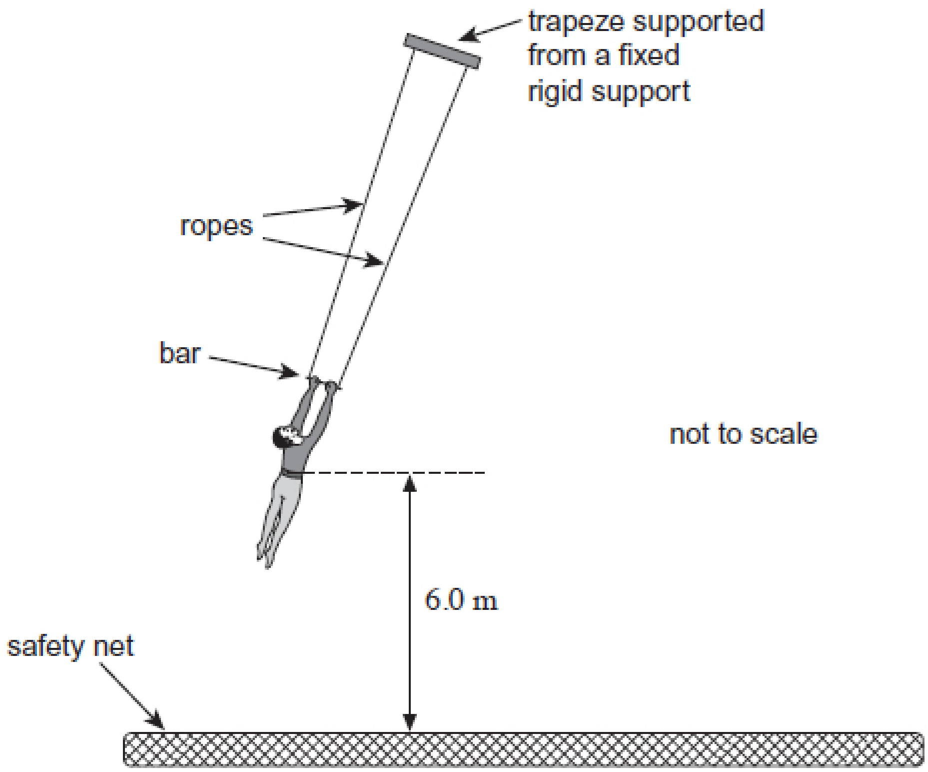
____

____

(2)
(Total 5 marks)

6 **Figure 1** shows an acrobat swinging on a trapeze.

Figure 1



(a) (i) State and explain how the tension in the ropes of the trapeze varies as the acrobat swings on the trapeze.

____

____

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____

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____

____

(3)

(ii) The period of the oscillation of the acrobat on the trapeze is 3.8 s.

Calculate the distance between the point of suspension of the trapeze and the centre of mass of the acrobat.

Assume that the acrobat is a point mass and that the system behaves as a simple pendulum.

distance ____________________ m

(2)

(b) At one instant the amplitude of the swing is 1.8 m. The acrobat lets go of the bar of the trapeze at the lowest point of the swing. He lands in a safety net when his centre of mass has fallen 6.0 m.

(i) Calculate the speed of the acrobat when he lets go of the bar.

speed ____________________ $m\ s^{-1}$

(3)

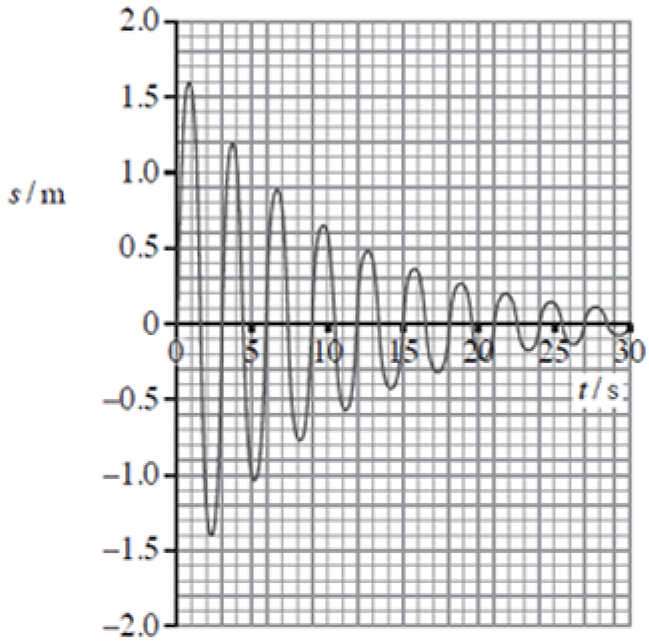
(ii) Calculate the horizontal distance between the point of suspension of the trapeze and the point at which the acrobat lands on the safety net.

horizontal distance ____________________ m

(3)

(c) **Figure 2** shows the displacement−time (s–t) graph for the bar of the trapeze after the acrobat has let go of the bar.

Figure 2



(i) Show that the amplitude of the oscillations decreases exponentially.

(3)

(ii) Explain why the period of the trapeze changes when the acrobat lets go of the bar.

____

____

____

____

(2)

(Total 16 marks)

7 (a) (i) Name the **two** types of potential energy involved when a mass–spring system performs vertical simple harmonic oscillations.

____

____

(1)

(ii) Describe the energy changes which take place during one complete oscillation of a vertical mass-spring system, starting when the mass is at its lowest point.

___

___

___

___

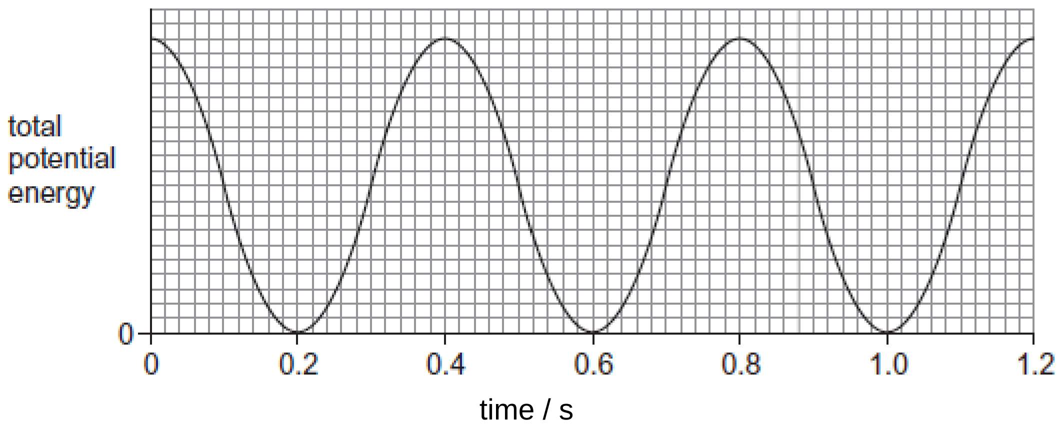
___

___

(2)

(b) **Figure 1** shows how the **total** potential energy due to the simple harmonic motion varies with time when a mass-spring system oscillates vertically.

Figure 1



(i) State the time period of the simple harmonic oscillations that produces the energy–time graph shown in **Figure 1**, explaining how you arrive at your answer.

___

___

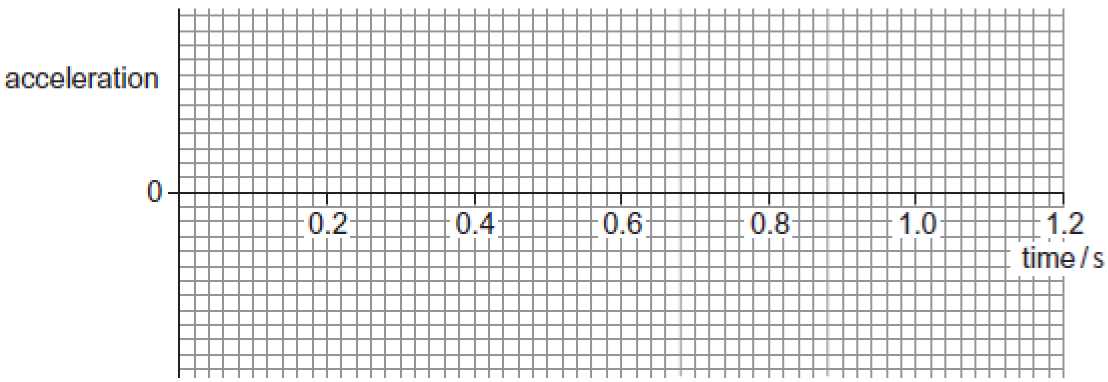
___

___

(2)

(ii) Sketch a graph on **Figure 2** to show how the acceleration of the mass varies with time over a period of 1.2 s, starting with the mass at the highest point of its oscillations. On your graph, upwards acceleration should be shown as positive and downwards acceleration as negative. Values are not required on the acceleration axis.

Figure 2



(2)

(c) (i) The mass of the object suspended from the spring in part (b) is 0.35 kg. Calculate the spring constant of the spring used to obtain **Figure 1**. State an appropriate unit for your answer.

spring constant ____________________ unit ___________

(3)

(ii) The maximum kinetic energy of the oscillating object is 2.0×10^{-2} J. Show that the amplitude of the oscillations of the object is about 40 mm.

(4)

(Total 14 marks)

8 (a) Give an equation for the frequency, f, of the oscillations of a simple pendulum in terms of its length, l, and the acceleration due to gravity, g.

State the condition under which this equation applies.

(2)

(b) The bob of a simple pendulum, of mass 1.2×10^{-2} kg, swings with an amplitude of 51 mm. It takes 46.5 s to complete 25 oscillations. Calculate

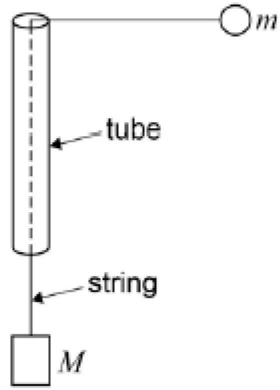
(i) the length of the pendulum,

(ii) the magnitude of the restoring force that acts on the bob when at its maximum displacement.

(6)

(Total 8 marks)

9 A string passes through a smooth thin tube. Masses m and M are attached to the ends of the string. The tube is moved so that the mass m travels in a horizontal circle of constant radius r and at constant speed v.



Which of the following expressions is equal to M ?

A $\frac{mv^2}{2r}$

B mv^2rg

C $\frac{mv^2}{rg}$

D $\frac{mv^2g}{r}$

(Total 1 mark)

10 The frequency of a body moving with simple harmonic motion is doubled. If the amplitude remains the same which of the following is also doubled?

A The time period.

B The total energy.

C The maximum velocity.

D The maximum acceleration.

(Total 1 mark)

11 A particle oscillates with undamped simple harmonic motion.

The acceleration of the particle

A is always in the opposite direction to its velocity.

B decreases as the potential energy increases.

C is proportional to the frequency.

D is least when the speed is greatest.

(Total 1 mark)

12 A simple pendulum and a mass-spring system have the same oscillation frequency f at the surface of the Earth. The pendulum and the mass-spring system are taken down a mine where the acceleration due to gravity is less than at the surface. What is the change in the frequency of the simple pendulum and the change in the frequency of the mass-spring system?

	simple pendulum	**mass-spring**	
A	f increases	f decreases	
B	f decreases	f decreases	
C	f increases	f stays unchanged	
D	f decreases	f stays unchanged	

(Total 1 mark)

13 What is the angular speed of a car wheel of diameter 0.400 m when the speed of the car is 108 km h^{-1}?

A 75 rad s^{-1}
B 150 rad s^{-1}
C 270 rad s^{-1}
D 540 rad s^{-1}

(Total 1 mark)

Mark schemes

1 (a) amplitude (of bob) is small
[**or** (angular) amplitude is less than or = 10°]
[**or** sin $\theta \approx \theta$ with θ explained] ✓

__or__ string is inextensible (or of negligible mass)

__or__ bob is a point mass

Ignore references to "air resistance".

1

(b) **The candidate's writing should be legible and the spelling, punctuation and grammar should be sufficiently accurate for the meaning to be clear.**

The candidate's answer will be assessed holistically. The answer will be assigned to one of three levels according to the following criteria.

High Level (Good to excellent): 5 or 6 marks
The information conveyed by the answer is clearly organised, logical and coherent, using appropriate specialist vocabulary correctly. The form and style of writing is appropriate to answer the question.

The candidate describes the arrangement of the apparatus clearly. They identify correctly the measurements to be made, and indicate how these measurements would be made. They describe a valid method by which a straight line graph may be obtained and show how g would be calculated from their graph. They are also aware of precautions that should be taken during the experiment to ensure that the result is accurate.

Intermediate Level (Modest to adequate): 3 or 4 marks
The information conveyed by the answer may be less well organised and not fully coherent. There is less use of specialist vocabulary, or specialist vocabulary may be used incorrectly. The form and style of writing is less appropriate.

The candidate is less clear about the experimental arrangement, gives a reasonable account of the measurements to be made and indicates a valid method by which a straight line graph may be obtained. They are less clear about how the result would be calculated from the graph, and they know the precautions less well.

Low Level (Poor to limited): 1 or 2 marks
The information conveyed by the answer is poorly organised and may not be relevant or coherent. There is little correct use of specialist vocabulary. The form and style of writing may be only partly appropriate.

The candidate gives a superficial account of the experimental arrangement, has some knowledge of the measurements to be made, but has only limited ability to show how a graphical method could be used to calculate the result. Some precautions may be known.

The description expected in a competent answer should include a coherent selection of the following points.

- Diagram or description showing a bob suspended from a fixed point, on which the length l may be labelled correctly.
- Length l of pendulum measured by ruler from fixed point of support to centre of mass of bob.
- Period T measured by stopwatch, by timing a number of oscillations.
- Measurement of T repeated for the same l and a mean value of T calculated.
- Measurements repeated for at least five different values of l.
- Graph of T^2 against l (or any other suitable linear graph) would be plotted.
- Graph is a straight line through origin, gradient is $4\pi2$/g (or correct expression for g from their graph).

Experimental measures such as the following are likely to be given:

- Small amplitude oscillations.
- Measure l to centre of mass of bob.

- Measure T from a large number of oscillations.
- Repeat timing for each length.
- Begin counting oscillations at nought when t = 0.
- Measure complete oscillations.
- Use of fiducial mark at centre of oscillations.
- Pendulum should swing in a vertical plane.
- Avoid very small values of l when repeating the experiment.

Credit may be given for any of these points which are described by reference to an appropriate labelled diagram.

*A **high level** answer must include*

1. a description of the apparatus,

2. a correct statement of the measurements to be made,

3. a correct graph plot,

4. a correct indication of how g would be found from the graph,

5. at least two precautions.

*An **intermediate level** answer must include (at least)*

*1 and 2, **or** 1 and 3, **or** 2 and 3, above and at least one precaution.*

*A **low level** answer must include (at least) any one of 1,2,3,4 above.*

*An inappropriate, irrelevant or physically incorrect answer should be awarded **a mark of zero**.*

*If the experiment described relates to a **compound pendulum**, mark to max 2.*

If a log graph is plotted and explained, it may gain credit.

*If a correct graph is **not** used, then maximum mark awarded is 3.*

max 6

(c) measured value of g will be 4× true value of g ✓

gradient of T^2 against l graph will be ¼ of expected value [**or** reference to g ∞ $1/T^2$ or equivalent] ✓

(T is halved so) T^2 is ¼ of true value ✓

2nd and 3rd marks may be covered by an analysis of the period equation.

3

[10]

2 (a) (grav) potential energy → kinetic energy → (grav) potential energy → kinetic energy → gravitational potential energy **(1)**

energy lost to surroundings in overcoming air resistance **(1)**

2

(b) (i) period $T = \left(\frac{42}{15}\right) = 2.8$ s **(1)**

use of $T = 2\pi\sqrt{\frac{l}{g}}$ gives length $l = \left(= \frac{T^2 g}{4\pi^2}\right) = \frac{2.8^2 \times 9.81}{4\pi^2}$ **(1)**

giving distance from pt of support to c of m, l = 1.9 (m)
or 1.95 (m) **(1)**

answer must be to **2 or 3 sf** only **(1)**

4

(ii) $E_k = mg\Delta h$ stated or used **(1)**

gives E_k of girl at lowest point = 18 × 9.81 × 0.25 = 44 (J) **(1)**

2

(iii) ½ mv^2 = 44.1 gives max speed of girl $v = \sqrt{\frac{2 \times 44.1}{18}}$ = 2.2 (m s^{-1}) **(1)**

[**alternatively**: A^2 = (3.9 – 0.25) × 0.25 gives A = 0.955 (m)
and v_{max} = 2π f A = (2π/2.8) × 0.955 = 2.1 (m s^{-1}) **(1)**]

1

(c) graph drawn on **Figure 2** which:

shows E_k = 0 at t = 0, T/2 and T **(1)**

has 2 maxima of similar size (some attenuation allowed)
at T/4 and 3T/4 **(1)**

is of the correct general shape **(1)**

3

[12]

3 (a) (mark should be at the equilibrium position) since this is where the mass moves with <u>greatest speed</u> [transit time is least] ✓

1

(b) (i) mean time for 20T (from sum of times ÷5) = 22.7 (s)$_1$ ✓
(minimum 3sf)

uncertainty (from half of the range) = 0.3 (s) $_2$ ✓ (accept trailing zeros here)

percentage uncertainty

$(\text{from } \frac{0.3}{22.7} \times 100)[\frac{100}{5} \times \sum \frac{0.3}{20T}] = 1.3\ (22)\%_3$ ✓

(allow full credit for conversion from 20T to T, e.g. 1.135 = $_1$✓
0.015 =$_2$ ✓ ecf for incorrect $_1$ ✓ and / or $_2$ ✓ earns $_3$ ✓

3

(ii) natural frequency (from $\frac{20}{22.7}$ and minimum 2 sf) = 0.88 (1)Hz [accept s^{-1}] ✓

(ecf for wrong mean 20*T*; accept ≥ 4 sf)

1

(c) (i) linear scale with at least 3 evenly-spaced convenient values (i.e. not difficult multiples) marked; the intervals between 1 Hz marks must be 40 ± 2 mm (100 ± 5 mm corresponds to 2.5 Hz) ✓

(ecf for wrong natural frequency: 100 ± 5 mm corresponds to $\frac{2.5f}{0.88}$ Hz)

1

(ii) 4 mm [allow ± 0.2 mm] ✓

1

(d) (i) student decreased intervals [smaller gaps] between [increase frequency / density of] readings (around peak / where A is maximum) ✓ ✓

[student took more / many / multiple readings (around peak) ✓]
(reject bland 'repeated readings' idea; ignore ideas about using data loggers with high sample rates)

2

(ii) new curve starting within ± 1 mm of A = 4 mm, f = 0 Hz with peak to right of that in Figure 3
(expect maximum amplitude shown to be less than for 2 spring system but don't penalise if this is not the case; likewise, the degree of damping need not be the same (can be sharper or less pronounced)
Peak at $\sqrt{2}$ value given in **(b)(ii)**; expect 1.25 Hz so peak should be directly over 50 ± 5 mm but take account of wrongly-marked scale ✓

2

[11]

4 (a) forced vibrations or resonance **(1)**

1

(b) reference to natural frequency (or frequencies) of structure **(1)**
driving force is at same frequency as natural frequency of structure **(1)**
resonance **(1)**
large <u>amplitude</u> vibrations produced or large energy transfer to structure**(1)**
could cause damage to structure [or bridge to fail] **(1)**

max 4

(c) stiffen the structure (by reinforcement) **(1)**
install dampers or shock absorbers **(1)**
[or other acceptable measure e.g. redesign to change natural frequency or increase mass of bridge or restrict number of pedestrians]

2

[7]

5 (a) vibrations are forced when periodic force is applied **(1)**

frequency determined by frequency of driving force **(1)**

resonance when frequency of applied force = natural frequency **(1)**

when vibrations of large amplitude produced

[or maximum energy transferred at resonance] **(1)**

(max 3)

(b) (i) damping when force opposes motion [or damping removes energy] **(1)**

(ii) damping reduces sharpness of resonance

[or reduces amplitude at resonant frequency] **(1)**

(2)

[5]

6 (a) (i) Tension minimum at extremities or maximum at middle / bottom

Tension depends on (component of) weight and required centripetal force / velocity

Increases as acrobat moves downwards

Tension at bottom = $mg + mv^2/r$ or Tension = weight + centripetal force

Tension at extremity = $mg/\cos\theta$ (θ is angle between rope and vertical)

Max 3

3

(ii) Use of $T = 2\pi\sqrt{(l/g)}$

3.6 (3.59) (m)

Allow for change of subject for use

2

(b) (i) Frequency of swing = 0.26 Hz

Use of $v = 2\pi fA$

3.0 or 2.97 (m s^{-1})

alternative method

Change in pe = gain in ke

Calculating Δh by geometry from swing = 0.48 m

3.1 or 3.06 (m s^{-1})

3

(ii) Use of $s = ½ at^2$

time to reach safety net = 1.11 s

s = their answer to **(b)(i)** × their time to reach the net = answer

(answer is 3.3 m if all correct)

Allow for change of subject for use

3

(c) (i) Attempt at valid test:

Fractional change in amplitude for same time interval

or use of 'half life' method

or use of exponential formula ($A = A_o e^{-kt}$) to show that k is constant

Correct calculation for one pair of amplitudes

Correct for second pair and conclusion

for half life method must see curve through peaks or other indication to find values between peaks

3

(ii) Period shorter

Centre of mass of trapeze artist was lower than the bar

Effective length of the pendulum is lower

Bar likely to be low mass now have a pendulum with distributed mass / no longer a simple pendulum / centre of mass is half way along suspending rope

Calculates new effective length of the pendulum (2 m)

Max 2

2

[16]

7 (a) (i) elastic potential energy **and** gravitational potential energy ✓

For elastic pe allow "pe due to tension", or "strain energy" etc.

1

(ii) elastic pe → kinetic energy → gravitational pe

→ kinetic energy → elastic pe ✓✓

[**or** pe→ke→pe→ke→pe is ✓ only]

[**or** elastic pe → kinetic energy → gravitational pe is ✓ only]

If kinetic energy is not mentioned, no marks.

Types of potential energy must be identified for full credit.

2

(b) (i) period = 0.80 s ✓

during one oscillation there are two energy transfer cycles

(or elastic pe→ke→gravitational pe→ke→elastic pe in 1 cycle)

or there are two potential energy maxima per complete oscillation ✓

Mark sequentially.

2

(ii) sinusoidal curve of period 0.80 s ✓

– cosine curve starting at t = 0 continuing to t = 1.2s ✓

For 1st mark allow ECF from T value given in (i).

2

(c) (i) use of $T = 2\pi\sqrt{\frac{m}{k}}$ gives $0.80 = 2\pi\sqrt{\frac{0.35}{k}}$ ✓

$\therefore k \left(= \frac{4\pi^2 \times 0.35}{0.80^2} \right)$ = 22 (21.6) ✓ N m^{-1} ✓

Unit mark is independent: insist on N m^{-1}.

Allow ECF from wrong T value from (i): use of 0.40s gives 86.4 (N m^{-1}).

3

(ii) maximum ke = (½ mv_{max}^2) = 2.0 × 10^{-2} gives

$v_{max}^2 = \frac{2.0\times10^{-2}}{0.5\times0.35}$ ✓ (= 0.114 m^2s^{-2}) and v_{max} = 0.338 (m s^{-1}) ✓

$v_{max} = 2\pi fA$ gives $A = \frac{0.338}{2\pi\times1.25}$ ✓

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm

[**or** maximum ke = (½ mv_{max}^2) = ½ $m\,(2\pi fA)^2$ ✓

½ × 0.35 × $4\pi^2$ × 1.25^2 × A^2 = 2.0 × 10^{-2} ✓

$\therefore A^2 = \frac{2\times2.0\times10^{-2}}{4\pi^2\times0.35\times1.25^2}$ ✓ (= 1.85 × 10^{-3})

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm]

[**or** maximum ke = maximum pe = 2.0 × 10^{-2} (J)

maximum pe = ½ $k\,A^2$ ✓

∴ 2.0 × 10^{-2} = ½ × 21.6 × A^2 ✓

from which $A^2 = \frac{2\times2.0\times10^{-2}}{21.6}$ ✓ (= 1.85 × 10^{-3})

and A = 4.3(0) × 10^{-2} m ✓ i.e. about 40 mm]

First two schemes include recognition that f = 1 / T
i.e. f = 1 / 0.80 = 1.25 (Hz).

Allow ECF from wrong T value from (i) – 0.40s
gives A = 2.15 × 10^{-2}*m but mark to max 3.*

Allow ECF from wrong k value from (i) –86.4Nm^{-1} *gives*
A = 2.15 × 10^{-2}*m but mark to max 3.*

4

[14]

8 (a) $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ **(1)**

Oscillations must be of small amplitude **(1)**

2

(b) (i) $f = \frac{25}{46.5} = 0.53(8)(s^{-1})$ **(1)**

[or $T = \frac{46.5}{25}$ = 1.8(6) (s)]

$l\left(=\frac{g}{4\pi^2 f^2}\right) = \frac{9.81}{4\pi^2 0.538^2}$ [or $l\left(=\frac{T^2 g}{4\pi^2}\right) = \frac{1.86^2\times9.81}{4\pi^2}$] **(1)**

l = 0.85(9)m **(1)**

(allow C.E. for values of f or T)

(ii) $a_{max}\{= (-)(2\pi f)^2 A\} = (2\pi \times 0.538)^2 \times 51 \times 10^{-3}$ **(1)**

$(= 0.583 \text{ ms}^{-2})$

(allow C.E. for value of f from (i))

$F_{max}(= ma_{max}) = 1.2 \times 10^{-2} \times 0.583$ **(1)**

$= 7.0 \times 10^{-3}\text{N}$ **(1)**
$(6.99 \times 10^{-3}\text{N})$

[or $F_{max}(= mg \sin \theta_{max})$ where $\sin \theta_{max} = \frac{51}{859}$ **(1)**

$= 1.2 \times 10^{-2} \times 9.81 \times \frac{51}{859}$ **(1)**

$= 6.99 \times 10^{-3}\text{N}$ **(1)**]

6

[8]

9 C

[1]

10 C

[1]

11 D

[1]

12 C

[1]

13 **B**

[1]

Examiner reports

1 In (a), quite apart from the obvious “small angle” limitation that applies when a simple pendulum is moving in simple harmonic motion, other answers which were accepted were an inextensible string, a string of negligible mass, or the bob having to behave as a point mass.

The experiment to determine the value of the Earth’s gravitational field strength by the use of a simple pendulum is well known, so in part (b) most students successfully described the experimental procedures they had carried out and the measures they had taken to produce an accurate result. When describing the experiment, some answers fell short of what was expected when describing the arrangement of the apparatus and many were quite obscure about the measurement of the length of the pendulum. Students had to give an acceptable graphical procedure in order to reach a high level assessment (5 or 6 marks). Any graph that would give a straight line through the origin was acceptable, but students had to show how they would arrive at the value of g from their graph. A few answers suggested that the time for *one oscillation only* would be found, hardly practical for an accurate result from this system. There were also some references to a pendulum that was clearly a *compound pendulum* rather than a simple pendulum; the maximum mark that could be given for these was 2.

Part (c) produced many good answers, in which it was clearly explained why the value of g obtained by the mistaken student would be four times the true value. It had been anticipated that the usual approach to the explanation would be by reference to the effect of the mistake on the gradient of the graph obtained in the experiment. In fact, more students preferred an approach relying on $g \propto 1/T^2$, which was equally acceptable. They usually recognised that T would be halved and that T^2 would be a quarter of its true value.

2 In part (a), the award of the full two marks was comparatively rare. Most answers were incomplete because candidates had not addressed the need to describe the energy changes of the bob '*over one complete oscillation, starting at its maximum displacement*'. A large proportion of candidates confined their attention to the first half of the oscillation, which limited them to half marks. Another error was a reluctance to refer to the potential energy as *gravitational*. Some candidates missed the point of the question completely, and wrote about velocity and acceleration in shm.

Calculation of the period of the swing in part (b) (i) was straightforward, and proved to be rewarding for most candidates. Those who confused period with frequency gained little credit, except for the mark for giving a final answer to an appropriate number of significant figures. Using the given data, the answer for the length was 1.948 m, when calculated to four significant figures. Final answers of 2.0 m (rather than 1.9 m) were therefore regarded as incorrect.

The solution to part (b) (ii), where the maximum E_k of the girl was needed, came readily from 'E_k gained = gravitational E_p lost'. Equating this result to ½ $m v^2$ then led to a neat solution to part (b) (iii), to find the maximum speed of the girl. Many candidates attempted much more tortuous routes to parts (ii) and/or (iii), using $v$max = 2 πfA. The principal downfall of this method (quite apart from its relative difficulty) was the adoption of 250 mm for the amplitude, A. Some successful solutions by the method were seen, however, where the correct value for A had been found by Pythagoras, or some equivalent calculation.

Many reasonable graphs were drawn in part (c), where the E_- against t graph was required, starting at maximum displacement. The majority of answers recognised that E_- would be zero at t = 0, T/2 and T. On most answers there were also correct maxima, of similar amplitude, within one square of T/4 and 3T/4 on the graph. The most demanding aspect was the shape of the graph; 'half wave rectified' waveforms tended to dominate, whilst triangular waveforms were by no means uncommon. Correct ($\sin^2$) shapes were comparatively rare, but credit was given for any shape which showed appropriately curved characteristics.

3 Encouraging numbers knew to say in **(a)** that the fiducial mark is placed at the equilibrium position because this is where ‘the transit time is least’ or ‘the speed of the mass is greatest’ but it was common to find statements such as ‘this is where the mass must pass through in every oscillation’ which attracted no credit.

Part **(b)(i)** and **(b)(ii)** were routine calculations in which candidates were usually successful although errors for the frequency arose due to rounding down earlier in the calculation. Full credit could be earned if the correct result for percentage uncertainty was reached using *T* rather than 20*T* but we withheld a mark in **(b)(ii)** if no unit was seen.

Part **(c)(i)** was very good and we took account of an incorrect **(b)(ii)** answer that led to an unexpected scale. We expected the scale to be linear and at least three convenient intervals marked. For the expected of 0.88 Hz result in **(b)(ii)** the 1 Hz intervals on the axis should have been 40 mm apart.

Many stated in **(c)(ii)** that the initial amplitude of X was 4 mm but a common error was to read off the amplitude at resonance (95 mm).

Part **(d)** was the most discriminating part of this question. In **(d)(i)** many grasped the idea behind the question but simply saying ‘take more readings’ was insufficient and only gained one mark. Some candidates avoided any ambiguity by saying ‘the intervals between the frequency readings around the peak were reduced’ to earn full credit. A common but unsuccessful idea was to assume a data logger was in use and to say that the sample rate was increased around the peak.

In **(d)(ii)** saw that removing one spring doubled the stiffness and increased the resonant frequency by a factor of $\sqrt{2}$. The new curve now had a resonant peak at about 1.25 Hz which for an axis with the expected calibration was 50 mm along the axis. Some spoiled their answer by starting the curve at 0 Hz with amplitude less than 4 mm.

Unlike last year the full mark range was utilised and the question discriminated very well. Those at the A/B boundary usually earned between 7 and 9 out of 11 and E/U candidates often scoring between 4 and 6.

4 Candidates' knowledge of forced vibrations, resonance and damping was rather better than might have been anticipated when this question was set. Consequently most candidates achieved high marks.

The Millennium footbridge has turned out to be a most complex structure, somewhat removed from the temporary "advancing army" type of footbridge traditionally considered in text books. When it was first opened, walkers on the bridge subconsciously adjusted their steps so that they were synchronised with lateral vibrations of the bridge span. In effect this was a feedback phenomenon in which the bridge vibrations were being passed back to the driving forces when there were a large number of people relative to the mass of the bridge and where the level of damping was low. Candidates could not be expected to be familiar with the full detail of this, and consequently the examiners took the view that "resonance" was an acceptable alternative to "forced vibrations" as an answer to part (a).

Part (b) was usually very rewarding. Clear reference to the frequency of the driving force was not always made, however. The examiners were also looking for mention of the amplitude of vibrations, rather than just the size of them. Many good, and some ingenious, answers were seen in part (c). The need for engineers to make the changes was not always appreciated. For example, some candidates suggested that the pedestrians should walk out of step. Neither were the limitations on the materials available to build a bridge readily recognised by some candidates; changing the natural frequency of the material of the bridge would not be a very practical solution. There were also candidates who thought that shortening the bridge would be an acceptable strategy! Despite these unsatisfactory responses, the majority of the candidates knew enough to suggest measures such as improved damping and a stiffer structure.

5 Answers to this question showed that even average candidates experienced difficulty in explaining themselves, and the weaker candidates struggled considerably. The knowledge that forced vibrations are caused by a driving force which is *periodic* was often omitted. Most candidates had a fair idea of resonance.

In part (b)(i) only a minority of candidates could explain that damping removes *energy* or occurs when the motion is opposed by a *force*. Very few candidates had the knowledge to score in part (b)(ii).

6 (a) (i) Although only 3 of 5 possible statements were required for full marks, many students found this question difficult. Some believed that the tension at the lowest point was mg, totally ignoring centripetal force. Some stated the weight acts at different angles as the trapeze swings, and some though the centripetal force was constant.

(ii) A few attempts to change the subject of the formula were incorrect, but most students were awarded full marks.

(b) (i) Correct answers were reached by those who used the formula $V = 2\pi fA$, but those that chose the energy method invariably used the wrong value of height (0.48m)

(ii) Most students had difficulty here. Some used the incorrect formula $x = A \cos (2\pi ft)$ and others used Pythagoras in an attempt to find the horizontal distance. Of those who chose an equation of motion, some chose inappropriately and were unable to find, the time (1.11 s).

(c) (i) Many students had little idea of how to tackle this question. The most straight forward correct method was to calculate the ratio of successive amplitudes. Those who attempted to find the half-life were expected to sketch the curve on the graph on (figure 2). The most difficult and rarely correctly executed method was to calculate k in = $A_o e^{-kt}$.

(ii) Most students completely missed the point that the simple pendulum is effectively shortened and so the period is reduced. Those using $T = 2\pi \sqrt{m/k}$ and others claiming that air resistance affected T gained no marks. There were lots of non-committal answers failing to state whether T increased or decreased.

7 Part (a)(i) offered an easy mark for naming the two types of potential energy involved in an oscillating suspended mass-spring system. “Gravitational potential energy” is clear and unambiguous, but a variety of terms appeared to be in use for the energy stored by a stretched spring. “Elastic potential energy” was the expected term, but “strain energy” was equally acceptable. For obvious reasons “stored energy” (when unqualified) was not.

Those who had concentrated on the wording of the question in part (a)(ii) – especially on “energy changes”, “one complete oscillation” and “starting at its lowest point” – were able to give good concise answers. Far too many of the students attempted to consider the absolute values of elastic and gravitational energies during an oscillation, which usually led them into confusion and irrelevance. Many answers stated that elastic potential energy would increase as the mass moved above the equilibrium position because the spring would be compressed. Inevitably a lot of answers described only half of an oscillation: if the energy types and changes were correctly described even this was given 1 mark. Answers which did not refer to the kinetic energy of the system were not credited.

In part (b)(i) those who appreciated that the total potential energy of the system passes through two maxima per oscillation, one at each amplitude, came up with the expected 0.8 s. Because they did not understand this, getting on for half of the students gave 0.4 s. There were consequences in the later parts of this question, where incorrect values from part (b)(i) were generally accepted as a basis for the work that followed. A small minority of the graphs drawn in part (b)(ii) were triangular, but the majority represented some form of sinusoidal variation. Whether this agreed with the expected period (0.8 s), and was a negative cosine curve, proved to be more testing issues.

The time period calculation in part (c)(i) was straightforward. This was rewarding for those who could substitute mass and period values correctly and then calculate the expected value. $N\ m^{-1}$ was the only answer accepted for the unit of *k*. The amplitude calculation in the final part of the question was often done well. Students who had made an error over the time period earlier were unable to show that the value of the amplitude would be about 40 mm, so were limited to a mark of 3 out of 4. Working from $T = 0.4$ s, many of these answers arrived at a value of 21.5 mm before the students introduced a mystery factor of 2 to end up with “about 40 mm”!

8 Appreciation that $T = 1/f$, and application of this to the familiar simple pendulum equation, readily gave the correct response to the initial requirement of part (a). Yet some candidates failed to score as a result of their failure to combine these two equations. The small angle condition under which the pendulum equation is valid was surprisingly poorly known, suggesting that many candidates never carry out any experiments in this area.

The first half of part (b) required candidates to determine the period (or frequency) from the data provided, and then to rearrange the pendulum equation to calculate its length. Algebraic rearrangement was a major source of difficulty for some candidates, whilst others failed to square the acceptable values that they had substituted. Each such error usually caused the loss of one mark. The second half was more demanding; solutions using either $F = m(2\pi f)^2 A$ or $mg \sin\theta_{max}$ were equally acceptable. A fairly common wrong approach was to suppose that $F = mv^2/r$ could be applied here.

13 This question, on angular speed, had also appeared in an earlier examination. The facility in 2016 was 62%, an improvement over the 56% facility last time. Over 20% of the students chose distractor A (75 rad s^{-1}) by confusing the diameter of the wheel with the radius.