(a) The graph shows how the gravitational potential varies with distance in the region above the surface of the Earth. $R$ is the radius of the Earth, which is 6400 km . At the surface of the Earth, the gravitational potential is $-62.5 \mathrm{MJ} \mathrm{kg}^{-1}$.


Use the graph to calculate
(i) the gravitational potential at a distance $2 R$ from the centre of the Earth,
(ii) the increase in the potential energy of a 1200 kg satellite when it is raised from the surface of the Earth into a circular orbit of radius $3 R$.
$\qquad$
$\qquad$
$\qquad$
(b) (i) Write down an equation which relates gravitational field strength and gravitational potential.
(ii) By use of the graph in part (a), calculate the gravitational field strength at a distance $2 R$ from the centre of the Earth.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(iii) Show that your result for part (b)(ii) is consistent with the fact that the surface gravitational field strength is about $10 \mathrm{~N} \mathrm{~kg}^{-1}$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

2 (a) (i) Explain what is meant by gravitational field strength.
$\qquad$
$\qquad$
$\qquad$
(ii) Describe how you would measure the gravitational field strength close to the surface of the Earth. Draw a diagram of the apparatus that you would use.
(b) (i) The Earth's gravitational field strength $(g)$ at a distance $(r)$ of $2.0 \times 10^{7} \mathrm{~m}$ from its centre is $1.0 \mathrm{~N} \mathrm{~kg}^{-1}$. Complete the table with the 3 further values of $g$.

| $g / \mathrm{N} \mathrm{kg}^{-1}$ | 1.0 |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $r / 10^{7} \mathrm{~m}$ | 2.0 | 4.0 | 6.0 | 8.0 |

(ii) Below is a grid marked with $g$ and $r$ values on its axes. Draw a graph showing the variation of $g$ with $r$ for values of $r$ between $2.0 \times 10^{7} \mathrm{~m}$ and $10.0 \times 10^{7} \mathrm{~m}$.

(iii) Estimate the energy required to raise a satellite of mass 800 kg from an orbit of radius $4.0 \times 10^{7} \mathrm{~m}$ to one of radius $10.0 \times 10^{7} \mathrm{~m}$.
(Total 14 marks)
3 (a) Explain why astronauts in an orbiting space vehicle experience the sensation of weightlessness.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(2)
(b) A space vehicle has a mass of 16800 kg and is in orbit 900 km above the surface of the Earth.

$$
\text { mass of the Earth }=5.97 \times 10^{24} \mathrm{~kg}
$$

radius of the Earth $=6.38 \times 10^{6} \mathrm{~m}$
(i) Show that the orbital speed of the vehicle is approximately $7400 \mathrm{~m} \mathrm{~s}^{-1}$.
(ii) The space vehicle moves from the orbit 900 km above the Earth's surface to an orbit 400 km above the Earth's surface where the orbital speed is $7700 \mathrm{~m} \mathrm{~s}^{-1}$.

Calculate the total change that occurs in the energy of the space vehicle.
Assume that the vehicle remains outside the atmosphere after the change of orbit. Use the value of $7400 \mathrm{~m} \mathrm{~s}^{-1}$ for the speed in the initial orbit.
$\qquad$ J

Figure 1 shows (not to scale) three students, each of mass 50.0 kg , standing at different points on the Earth's surface. Student $\mathbf{A}$ is standing at the North Pole and student $\mathbf{B}$ is standing at the equator.

Figure 1

Figure 2
not to scale


The radius of the Earth is 6370 km .
The mass of the Earth is $5.98 \times 10^{22} \mathrm{~kg}$.
In this question assume that the Earth is a perfect sphere.
(a) (i) Use Newton's gravitational law to calculate the gravitational force exerted by the Earth on a student.
force $\qquad$ N
(ii) Figure 2 shows a closer view of student $\mathbf{A}$.

Draw, on Figure 2, vector arrows that represent the forces acting on student $\mathbf{A}$.
(b) (i) Show that the linear speed of student $\mathbf{B}$ due to the rotation of the Earth is about 460 $\mathrm{ms}^{-1}$.
(ii) Calculate the magnitude of the centripetal force required so that student $\mathbf{B}$ moves with the Earth at the rotational speed of $460 \mathrm{~ms}^{-1}$.
magnitude of the force $\qquad$ N
(iii) Show, on Figure 1, an arrow showing the direction of the centripetal force acting on student C.
(c) Student B stands on a bathroom scale calibrated to measure weight in newton ( N ). If the Earth were not rotating, the weight recorded would be equal to the force calculated in part (a)(i).

State and explain how the rotation of the Earth affects the reading on the bathroom scale for student B.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

The graph below shows how the gravitational potential energy, $E_{p}$, of a 1.0 kg mass varies with distance, $r$, from the centre of Mars. The graph is plotted for positions above the surface of Mars.

(a) Explain why the values of $E_{p}$ are negative.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) Use data from the graph to determine the mass of Mars.
mass of Mars $\qquad$ kg
(c) Calculate the escape velocity for an object on the surface of Mars.
escape velocity $\qquad$ $\mathrm{m} \mathrm{s}^{-1}$
(d) Show that the graph data agree with $E_{\mathrm{p}} \propto \frac{1}{r}$
(a) (i) State the relationship between the gravitational potential energy, $E_{p}$, and the gravitational potential, $V$, for a body of mass $m$ placed in a gravitational field.
$\qquad$
$\qquad$
(ii) What is the effect, if any, on the values of $E_{\mathrm{p}}$ and $V$ if the mass $m$ is doubled? value of $E_{\mathrm{p}}$ $\qquad$ value of $V$ $\qquad$
(b)


The diagram above shows two of the orbits, $\mathbf{A}$ and $\mathbf{B}$, that could be occupied by a satellite in circular orbit around the Earth, $\mathbf{E}$.
The gravitational potential due to the Earth of each of these orbits is:
orbit A $\quad$ - 12.0 $\mathrm{MJ} \mathrm{kg}^{-1}$
orbit B $\quad-36.0 \mathrm{MJ} \mathrm{kg}^{-1}$.
(i) Calculate the radius, from the centre of the Earth, of orbit A.
answer =
$\qquad$ m
(ii) Show that the radius of orbit B is approximately $1.1 \times 10^{4} \mathrm{~km}$.
(iii) Calculate the centripetal acceleration of a satellite in orbit B.
$\qquad$ $\mathrm{m} \mathrm{s}^{-2}$
(iv) Show that the gravitational potential energy of a 330 kg satellite decreases by about 8 GJ when it moves from orbit $\mathbf{A}$ to orbit $\mathbf{B}$.
(c) Explain why it is not possible to use the equation $\Delta E_{\mathrm{p}}=m g \Delta h$ when determining the change in the gravitational potential energy of a satellite as it moves between these orbits.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

7 The gravitational field strength at the surface of a planet, X , is $19 \mathrm{Nkg}^{-1}$.
(a) (i) Calculate the gravitational potential difference between the surface of $X$ and a point 10 m above the surface, if the gravitational field can be considered to be uniform over such a small distance.
$\qquad$
$\qquad$
(ii) Calculate the minimum amount of energy required to lift a 9.0 kg rock a vertical distance of 10 m from the surface of X .
$\qquad$
$\qquad$
(iii) State whether the minimum amount of energy you have found in part (ii) would be different if the 9.0 kg mass were lifted a vertical distance of 10 m from a point near the top of the highest mountain of planet $X$. Explain your answer.
$\qquad$
$\qquad$
$\qquad$
(b) Calculate the gravitational field strength at the surface of another planet, Y , that has the same mass as planet X , but twice the diameter of X .
$\qquad$
$\qquad$
$\qquad$
$\qquad$

8 (a) State Newton's law of gravitation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(b) In 1798 Cavendish investigated Newton's law by measuring the gravitational force between two unequal uniform lead spheres. The radius of the larger sphere was 100 mm and that of the smaller sphere was 25 mm .
(i) The mass of the smaller sphere was 0.74 kg . Show that the mass of the larger sphere was about 47 kg .
density of lead $=11.3 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$
(ii) Calculate the gravitational force between the spheres when their surfaces were in contact.
$\qquad$ N
(c) Modifications, such as increasing the size of each sphere to produce a greater force between them, were considered in order to improve the accuracy of Cavendish's experiment. Describe and explain the effect on the calculations in part (b) of doubling the radius of both spheres.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

9 A satellite X is in a circular orbit of radius $r$ about the centre of a spherical planet of mass $M$.


Which line, A to $\mathbf{D}$, in the table gives correct expressions for the centripetal acceleration $a$ and the speed $v$ of the satellite?

|  | Centripetal acceleration $\boldsymbol{a}$ | Speed $\boldsymbol{v}$ |
| :---: | :---: | :---: |
| A | $\frac{G M}{2 r}$ | $\sqrt{\frac{G M}{2 r}}$ |
| B | $\frac{G M}{2 r}$ | $\sqrt{\frac{G M}{r}}$ |
| C | $\frac{G M}{r^{2}}$ | $\sqrt{\frac{G M}{2 r}}$ |
| D | $\frac{G M}{r^{2}}$ | $\sqrt{\frac{G M}{r}}$ |

A satellite orbiting the Earth moves to an orbit which is closer to the Earth.
Which line, A to D, in the table shows correctly what happens to the speed of the satellite and to the time it takes for one orbit of the Earth?

|  | Speed of satellite | Time For One Orbit Of Earth |
| :---: | :---: | :---: |
| A | decreases | decreases |
| B | decreases | increases |
| C | increases | decreases |
| D | increases | increases |

11 A spacecraft of mass $m$ is at the mid-point between the centres of a planet of mass $M_{1}$ and its moon of mass $M_{2}$. If the distance between the spacecraft and the centre of the planet is $d$, what is the magnitude of the resultant gravitational force on the spacecraft?

A $\frac{G m\left(M_{1}-M_{2}\right)}{d}$
B $\frac{G m\left(M_{1}+M_{2}\right)}{d^{2}}$
C $\frac{\operatorname{Gm}\left(M_{1}-M_{2}\right)}{d^{2}}$
D $\frac{G m\left(M_{1}+M_{2}\right)}{d}$

12 Which one of the following statements about gravitational potential is correct?
A gravitational potential can have a positive value
B the gravitational potential at the surface of the Earth is zero
C the gravitational potential gradient at a point has the same numerical value as the gravitational field strength at that point

D the unit of gravitational potential is $\mathrm{Nkg}^{-1}$
(Total 1 mark)
13 When a space shuttle is in a low orbit around the Earth it experiences gravitational forces $F_{E}$ due to the Earth, $F_{\mathrm{M}}$ due to the Moon and $F_{\mathrm{S}}$ due to the Sun. Which one of the following correctly shows how the magnitudes of these forces are related to each other?
mass of Sun $=1.99 \times 10^{30} \mathrm{~kg}$
mass of Moon $=7.35 \times 10^{22} \mathrm{~kg}$
mean distance from Earth to Sun $=1.50 \times 10^{11} \mathrm{~m}$
mean distance from Earth to Moon $=3.84 \times 10^{8} \mathrm{~m}$
A $\quad F_{\mathrm{E}}>F_{\mathrm{S}}>F_{\mathrm{M}}$
B $\quad F_{S}>F_{E}>F_{M}$
C $\quad F_{E}>F_{M}>F_{S}$
D $\quad F_{M}>F_{E}>F_{S}$
(Total 1 mark)

14 The gravitational field strengths at the surfaces of the Earth and the Moon are $9.8 \mathrm{~N} \mathrm{~kg}^{-1}$ and $1.7 \mathrm{~N} \mathrm{~kg}^{-1}$ respectively. If the mass of the Earth is $81 \times$ the mass of the Moon, what is the ratio of the radius of the Earth to the radius of the Moon?

A 3.7
B 5.8
C 14
D 22
(Total 1 mark)
15 Two stars of mass $M$ and $4 M$ are at a distance $d$ between their centres.


The resultant gravitational field strength is zero along the line between their centres at a distance $y$ from the centre of the star of mass $M$.

What is the value of the ratio $\frac{y}{d}$ ?
A $\frac{1}{2}$
B $\frac{1}{3}$
C $\frac{2}{3}$
D $\frac{3}{4}$
(Total 1 mark)

## Mark schemes

(a) (i) $-31 \mathrm{MJ} \mathrm{kg}^{-1}$ (1)
(ii) increase in potential energy $=m \Delta V(1)$

$$
\begin{aligned}
& =1200 \times(62-21) \times 10^{6}(1) \\
& =4.9 \times 10^{10} \mathrm{~J}(1)
\end{aligned}
$$

(b) (i) $g=-\frac{\Delta V}{\Delta x}$ (1)
(ii) $g$ is the gradient of the graph $=\frac{62.5 \times 10^{6}}{4 \times 6.4 \times 10^{6}}$ (1)

$$
=2.44 \mathrm{~N} \mathrm{~kg}^{-1}(1)
$$

(iii) $\quad g \propto \frac{1}{R^{2}}$ and $R$ is doubled (1)
expect $g$ to be $\frac{9.81}{4}=2.45 \mathrm{~N} \mathrm{~kg}^{-1}$ (1)
[alternative (iii)
$g \propto \frac{1}{R^{2}}$ and $R$ is halved (1)
expect $g$ to be $2.44 \times 4=9.76 \mathrm{~N} \mathrm{~kg}^{-1}$ (1)]

2 (a) (i) force per unit mass (allow equation with defined terms)
(ii) diagram of method that will work (pendulum / light gates / solenoid and mechanical gate / strobe photography / video)
pair of measurements (eg length of pendulum and (periodic) time / distance and time of fall - could be shown on diagram)
instruments to measure named quantities (may be on diagram)
correct procedure (eg calculate period for range of lengths, measure the time of fall for range of heights)
good practice - series of values and averages / use of gradient of graph
appropriate formula and how $g$ calculated
(b) (i) evidence of $g r^{2}$ being used
values of $0.25,0.11,0.06(25)$
no s.f. penalty here unless values given as fractions
(ii) points correctly plotted on grid (e.c.f.)
smooth curve of high quality at least to $10 \times 10^{7} \mathrm{~m}$, no intercept on $r$ axis
(iii) attempt to use area under curve
$\begin{array}{ll}\text { evidence of } \times 800 \mathrm{~kg} & \text { B1 } \\ \text { B1 }\end{array}$
$(4.3-5.3) \times 10^{9} \mathrm{~J}$
or
use of equation for potential $\Delta E_{G}=m\left(g_{1} r_{1}-g_{2} r_{2}\right)$
evidence of $\times 800 \mathrm{~kg}$
$(4.7-4.9) \times 10^{9} \mathrm{~J}$
max 2 if assumed values of $G$ and $M$ used
allow calculation of $G M$ from graph followed by substitution into $\Delta E_{G}=M_{G}(\mathrm{~m} /$ $r_{1}-m / r_{2}$ ) for 3 marks

3 (a) Idea that both astronaut and vehicle are travelling at same (orbital) speed or have the same (centripetal) acceleration / are in freefall

Not falling at the same speed

No (normal) reaction (between astronaut and vehicle)
(b) (i) Equates centripetal force with gravitational force using appropriate formulae
E.g. $\frac{G M m}{r^{2}}=\frac{m v^{2}}{r}$ or $m r \omega^{2}$

Correct substitution seen e.g. $v^{2}=\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{\text { anyvalue of radius }}$
(Radius of) $7.28 \times 10^{6}$ seen or $6.38 \times 10^{6}+0.9 \times 10^{6}$
$7396\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ to at least 4 sf
Or v ${ }^{2}=5.47 \times 10^{7}$ seen
(ii) $\quad \triangle P E=6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4}(1 /(7.28 \times$ $\left.\left.10^{6}\right)-1 /\left(6.78 \times 10^{6}\right)\right)$

C1
$-6.8 \times 10^{10} \mathrm{~J}$
C1
$\Delta K E=0.5 \times 1.68 \times 10^{4} \times\left(7700^{2}-7400^{2}\right)=3.81 \times 10^{10} \mathrm{~J}$

C1
$\Delta K E-\Delta P E=(-) 2.99 \times 10^{10}(\mathrm{~J})$
A1
OR
Total energy in original orbit shown to be (-)GMm / $2 r$ or $m v^{2} / 2-G M m / r$

C1
Initial energy
$=-6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4} /\left(2 \times 7.28 \times 10^{6}\right)$ $=4.59 \times 10^{11}$

C1
Final energy
$=-6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.68 \times 10^{4} /\left(2 \times 6.78 \times 10^{6}\right)$
$=4.93 \times 10^{11}$
$3.4 \times 10^{10}(\mathrm{~J})$
Condone power of 10 error for $C$ marks
A1
4
[10]
(a) (i) Use of $F-G M m / r^{2}$

Allow 1 for
-correct formula quoted but forgetting square in substitution

Correct substitution of data
M1
-missing $m$ in substitution
491 (490)N
A1
-substutution with incorrect powers of 10
Condone 492 N ,
(ii) Up and down vectors shown (arrows at end) with labels

B1
allow $\mathrm{W}, \mathrm{mg}$ (not gravity); $\quad R$
allow if slightly out of line / two vectors
shown at feet
up and down arrows of equal lengths
B1
condone if colinear but not shown acting on body
In relation to surface $W \leq R$ (by eye) to allow for weight vector starting in middle of the body
Must be colinear unless two arrows shown in which case $R$ vectors $1 / 2$ W vector(by eye)
(b) (i) Speed $=2 \pi r / T$

B1
Max 2 if not easy to follow
$2 \pi 6370000 /(24 \times 60 \times 60)$
B1
$463 \mathrm{~m} \mathrm{~s}^{-1}$
B1
Must be 3sf or more
(ii) Use of $F=m v^{2} / r$

C1
Allow 1 for use of $F=m r \omega^{2}$ with $\omega=460$
(iii) Correct direction shown
(Perpendicular to and toward the axis of rotation)
NB - not towards the centre of the earth
B1
(c) Force on scales decreases / apparent weight decreases

Appreciates scale reading = reaction force
C1
The reading would become 489 (489.3)N or reduced by 1.7 N )
A1
Some of the gravitational force provides the necessary centripetal force
B1

$$
\text { or } R=m g-m v^{2} / r
$$

5 (a) zero potential at infinity (a long way away)
energy input needed to move to infinity (from the point) work done by the field moving object from infinity potential energy falls as object moves from infinity

B1
2
(b) Any pair of coordinates read correctly

$$
\begin{aligned}
& \pm 1 / 2 \text { square } \\
& \text { Use of } E_{p \text { or }} V=(-) \frac{G M}{r} \\
& \text { Rearrange for } M \\
& 6.4( \pm 0.5) \times 10^{23} \mathrm{~kg}
\end{aligned}
$$

A1
3
(c) Reads correct potential at surface of Mars $=-12.6(\mathrm{MJ})$

C1
or reads radius of mars correctly $\left(3.5 \times 10^{6}\right)$
equates to $1 / 2 \mathrm{v}^{2}$ (condone power of 10 in MJ )
C1
use of $v=\sqrt{ }(2 G M / r)$ with wrong radius
$5000 \pm 20 \mathrm{~m} \mathrm{~s}^{-1}$ (condone 1sf e.g. $5 \mathrm{~km} \mathrm{~s}^{-1}$ )
A1
e.c.f. value of $M$ from (b) may be outside range for other method 6.2 $\times 10^{-9} \times \sqrt{ }$ their M
(d) Attempts 1 calculation of Vr

B1
Many values give 4.2.... so allow mark is for reading and using correct coordinates but allow minor differences in readings Ignore powers of 10 but consistent

Two correct calculation of $V r$

Three correct calculations with conclusion
B1

6 (a) (i) relationship between them is $E_{\mathrm{p}}=m V$ (allow $\Delta E_{\mathrm{p}}=m \Delta V$ [or $V$ is energy per unit mass (or per kg)] (1)
(ii) value of $E_{p}$ is doubled (1)
value of $V$ is unchanged (1)
(b) (i) use of $V=-\frac{G M}{r}$ gives $r_{A}=\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{12.0 \times 10^{6}}$ (1)

$$
=3.3(2) \times 10^{7}(\mathrm{~m})(1)
$$

(ii) since $V \propto(-) \frac{1}{r}\left(\operatorname{or} \frac{r_{A}}{r_{B}}=\frac{v_{E}}{v_{A}}=\frac{36.0}{12.0}=3\right) r_{E}=\frac{3.32 \times 10^{7} \mathrm{~m}}{3}$ (1) $\left(\right.$ which is $\left.\approx 1.1 \times 10^{4} \mathrm{~km}\right)$
(iii) centripetal acceleration $g_{B}=\frac{G M}{r_{E}{ }^{2}}=\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{\left(1.11 \times 10^{7}\right)^{2}}$
[allow use of $1.1 \times 10^{7} \mathrm{~m}$ from (b)(ii)]
$=3.2\left(\mathrm{~m} \mathrm{~s}^{-2}\right)$
[alternatively, since $g_{\mathrm{B}}=(-) \frac{v_{B}}{v_{a}}, g_{E}=\frac{36.0 \times 10^{6}}{1.11 \times 10^{7}}$

$$
\begin{equation*}
\left.=3.2\left(\mathrm{~m} \mathrm{~s}^{-2}\right)(1)\right] \tag{1}
\end{equation*}
$$

(iv) use of $\Delta E_{\mathrm{p}}=m \Delta V$ gives $\Delta E_{\mathrm{p}}=330 \times(-12.0-(-36.0)) \times 10^{6}(1)$
(which is $7.9 \times 10^{9} \mathrm{~J}$ or $\approx 8 \mathrm{GJ}$ )
(c) $g$ is not constant over the distance involved
(or $g$ decreases as height increases
or work done per metre decreases as height increases
or field is radial and/or not uniform) (1)
(a) (i) $\left(g=-\frac{\Delta V}{\Delta x}\right) 19=(-) \frac{\Delta V}{10}$ gives $\Delta V=190$ (1) $\mathrm{J} \mathrm{kg}^{-1}$ (1)
(ii) $W(=m \Delta V)=9.0 \times 190=1710 \mathrm{~J}[\mathrm{or} m g h=9.0 \times 19 \times 10=1710 \mathrm{~J}]$ (1)
(iii) on mountain, required energy would be less because gravitational field strength is less (1)
$\max 3$
(b) $\quad g \propto \frac{1}{r^{2}}\left(\right.$ or $\mathrm{F} \propto \frac{1}{r^{2}}$ or correct use of $F=\frac{G M m}{r^{2}}$ )
$\therefore g^{\prime}=\frac{19}{2^{2}}=4.75\left(\mathrm{Nkg}^{-1}\right)(1)$
(a) force of attraction between two point masses (or particles)
(1) proportional to product of masses (1)
inversely proportional to square of distance between them (1)

## [alternatively

quoting an equation, $F=\frac{G M_{1} M_{2}}{r^{2}}$ with all terms defined (1)
reference to point masses (or particles) or $r$ is distance between centres (1)
$F$ identified as an attractive force (1)]
$\max 2$
(b) (i) mass of larger sphere $M_{\mathrm{L}}\left(=\frac{4}{3} \pi r^{3} \rho\right)=\frac{4}{3} \pi \times(0.100)^{3} \times 11.3 \times 10^{3}$ (1) $=47(.3)(\mathrm{kg})(1)$
[alternatively

$$
\begin{aligned}
& \text { use of } M \mu r^{3} \text { gives } \frac{M_{\mathrm{L}}}{0.74}=\left(\frac{100}{25}\right)^{3}(1)(=64) \\
& \text { and } \left.M_{\mathrm{L}}=64 \times 0.74=47(.4)(\mathrm{kg})(1)\right]
\end{aligned}
$$

(c) for the spheres, mass $\mu$ volume (or $\mu r^{3}$, or $M=\frac{4}{3} \pi r^{3} \rho$ )
mass of either sphere would be $8 \times$ greater ( $378 \mathrm{~kg}, 5.91 \mathrm{~kg}$ ) (1)
this would make the force $64 \times$ greater (1)
but separation would be doubled causing force to be $4 \times$ smaller (1)
net effect would be to make the force $(64 / 4)=16 \times$ greater (1) (ie $2.38 \times 10^{-6} \mathrm{~N}$ )


12 C
13 A
14 A
[1]
15 B

## Examiner reports

There were a few problems with reading the graph scales in part (a)(i) but most candidates gave an acceptable answer in the range between 30 and $32 \mathrm{Mj} \mathrm{kg}{ }^{-1}$. Many candidates seem to have found the understanding of gravitational potential to be a considerable obstacle, and most were unable to use the graph in the intended way when attempting parts (a)(ii) and (b)(ii). In part (a)(ii) it was common for candidates to use $\Delta \mathrm{E}_{\mathrm{p}}=m g \Delta h$, or to attempt to calculate $V$ from $V=-(G M /$ $r)$ with $M$ as 1200 kg .

In part (b) the equation $g=-(\Delta V / \Delta x)$ was known by only some candidates, and even they were rarely able to identify it with the gradient of the graph, as required in part (b)(ii). Tangents to the curve at a distance of $2 R$ were hardly ever drawn, and candidates tended to try a variety of alternative routes to arrive at $g=2.4 \mathrm{~N} \mathrm{~kg}-1$. In part (b)(iii) it was expected that a knowledge of $g$ $\propto\left(1 / R^{2}\right)$ would enable candidates to spot that halving $R$ would quadruple $g$. Most candidates preferred to recalculate a surface value for $g$ by using $g=-\left(G M / R^{2}\right)$ which was not what had been asked in the question.
(a) Most candidates mentioned the lack of reaction force but some answers were spoilt by claiming that there was no resultant force or even no gravity.
(b) (i) Each step needed to be clearly shown, starting with the statement that gravitational force $=$ centripetal force.

There were several cases of the use of 900 km for the radius.
(ii) Common errors were treating potential energy as positive, the use of the wrong radii and the use of mgh.
(a) (i) Most candidates were able to make good progress with this calculation and there were many correct answers.
(ii) Many attempts were unconvincing and frequently carelessly drawn. Weight and reaction forces were often shown as not being collinear. Some showed a reaction force at one of the feet but not the other. That the length of a vector should represent magnitude was not realised by many candidates.
(b) (i) A good proportion of correct approaches were seen but many candidates seemed unsure what equation to use so quoted some that were not relevant. Good structure in a mathematical argument is an important skill in all problems but even more so in 'show that' type questions were marks are awarded for each step.
(ii) Again there was a good proportion of correct response. Some candidates used $F=$ $m r \omega^{2}$; and $460 \mathrm{~m} \mathrm{~s}^{-1}$ for $\omega$.
(iii) Misunderstanding about centripetal force was common here and there were relatively few correct answers. The majority showed the force acting toward the centre of the Earth. Whilst a component of this force provides the centripetal force, the direction of the centripetal force is toward the centre of rotation which in the diagram is perpendicular and toward the axis of rotation of the Earth.
(c) There were very good answers from candidates who understood that the scales read the reaction force. There were many who knew the formula $m g-R=m v^{2} / r$ but thought that the scales would record $m g$ and assumed $R$ to remain constant so that the centripetal force increased the scale reading.
(a) Most candidates knew the infinity reference point but many had difficulty explaining the negative sign. A statement such as 'work has to be done to move a mass from infinity' begs the question whether work being done within the Earth-mass system energy by the gravitational field to reduce the potential energy or whether the work is being done by an outside energy source.
(b) A high proportion of candidates completed this successfully either using the potential formula or by considering potential energy changes. Use of the force formula was a common error.
(c) Most read the radius from the graph and used the value of mass from (b) rather than the method of reading the potential energy ( - ) 12.6 MJ from the graph and equating it to $1 / 2 \mathrm{v}^{2}$. Using arbitrary energies or incorrect reading from the graph were common errors.
(d) The majority appreciated what was an acceptable test and almost half the candidates scored full marks. Others usually used only two points on the curve or used three appropriately but left it to the examiner to make the conclusion from the values obtained.

Many very good answers were seen in part (a) (i), expressed either fully in words or simply by quoting $E_{\mathrm{p}}=m V$. The corresponding equation for an incremental change, $\Delta E_{\mathrm{p}}=m \Delta V$, was also acceptable but mixed variations on this such as $E_{\mathrm{p}}=m \Delta V$ (which showed a lack of understanding) were not. The consequences of doubling $m$ were generally well understood in part (a) (ii), where most candidates scored highly, but some inevitably thought that $E_{\mathrm{p}}$ would be unchanged whilst $V$ would double.

Candidates who were not fully conversant with the metric prefixes used with units had great difficulty in part (b), where it was necessary to know that $1 \mathrm{MJ}=10^{6} \mathrm{~J}, 1 \mathrm{GJ}=10^{9} \mathrm{~J}$, and (even) 1 $\mathrm{km}=10^{3} \mathrm{~m}$. Direct substitution into $V=(-) G M / r$ (having correctly converted the value of $V$ to J $\mathrm{kg}^{-1}$ ) usually gave a successful answer for the radius of orbit $\mathbf{A}$ in part (b) (i). A similar approach was often adopted in part (b) (ii) to find the radius of orbit B, although the realisation that $V \propto 1 / r$ facilitated a quicker solution. Some candidates noticed that $V_{B}=3 V_{A}$ and guessed that $r_{B}=r_{A} / 3$, but this was not allowed when there was no physical reasoning to support the calculation.

Part (b) (iii) caused much difficulty, because candidates did not always appreciate that the centripetal acceleration of a satellite in stable orbit is equal to the local value of $g$, which is equal to $G M / r^{2}$. This value turns out to equal to $V / r$, which provided an alternative route to the answer. Many incredible values were seen, some of them greatly exceeding $9.81 \mathrm{~m} \mathrm{~s}^{-2}$.

Part (c) was generally well understood, with some very good and detailed answers from the candidates. Alternative answers were accepted: either that $g$ is not constant over such large distances, or that the field of the Earth is radial rather than uniform.

The question was intended to direct the candidates towards $g=-(\Delta V / \Delta x)$ in part (a), but some preferred to resort to $V=-(G M / r)$. The amalgamation of this equation with $\left(G M / r^{2}\right)=19$ and $r=10$ led to an apparently correct answer of $190 \mathrm{~J} \mathrm{~kg}^{-1}$. However, the physics of this approach is wrong, because the distance from the surface of planet $X$ to its centre is certainly not 10 m .
Therefore no credit was given unless $g=-(\Delta V / \Delta x)$ was the basis of the solution. It was seldom possible to award the separate mark allocated for the correct unit of gravitational potential difference, because very few candidates knew that it is $\mathrm{Jkg}^{-1}$. The straightforward approach to part (a)(ii) is through the direct application of $m \Delta V$, but most candidates preferred to use $m g \Delta h$ which was equally acceptable. Most candidates recognised that the field strength would be slightly less at the top of the highest mountain of $X$ and therefore gave an acceptable response in part (a)(iii).

In part (b) the ability to distinguish between $2 r^{2}$ and $(2 r)^{2}$ was beyond the mathematical skills of the many candidates who, having realised that $g \propto\left(1 / r^{2}\right)$. gave an incorrect answer of 9.5 N $\mathrm{kg}^{-1}$.

Many correct statements of Newton's law of gravitation were seen in part (a). Some candidates referred to just one aspect of the law ( $\propto M_{1} M_{2}$, or $\propto 1 / r^{2}$, not both together) and only received one mark. A reference to point masses - which helps when explaining the meaning of $r$ - was not common. In fact a clear understanding of the meaning of $r$ was expected in satisfactory answers. The common inadequate responses, when neither was more fully explained, were 'radius' and 'distance' Candidates who tried to rely simply on quoting $F=G M_{1} M_{2} / r^{2}$ were awarded a mark only when the terms in the equation were correctly identified; a further mark was available to them if they gave a clear definition of $r$ or referred to the nature of the force as attractive.

Part (b) (i) could be approached using either 'mass = volume $\times$ density' or 'mass $\propto r^{3}$. The first method was far more common, and most answers were satisfactory. On this paper, this was the first example of a question requiring candidates to 'show that...' Convincing answers to this type of question should include the fullest possible working, in which the final answer is quoted to one more significant figure than the value given in the question. Here, for example, a value of 47.3 kg was convincing. Part (b) (ii) also proved to be very rewarding for most candidates.

Common errors here were failing to square the denominator, or to assume that surfaces in contact meant that $r=0$ (whilst still arriving at a finite numerical answer!).

Whilst many correct and well argued answers were seen in part (c), it was clear that some candidates had not read the question with sufficient care. Two requirements for a satisfactory answer ought to be clear from the wording of the question: the need to give a quantitative answer, and to confine the answer to the effect on the calculations in part (b). 'Calculations' (plural) was a strong hint that the mass of both spheres would be affected, but there were many answers in which it was assumed that the masses would not be changed. This meant that a maximum mark of 1 out of 4 could be awarded, for the $1 / r^{2}$ relationship alone. The incorrect use of language sometimes also limited the mark that could be awarded for the answers here: candidates who stated that doubling the separation would reduce the force 'by one quarter' could not be credited with a mark.

This question was about satellites. The former required correct algebraic expressions for the centripetal acceleration and speed of a satellite in circular orbit around a planet. Just over four-fifths of the responses were correct.

This question tested students' understanding of the effect of the descent towards a planet on the speed and orbit time. $74 \%$ of them knew that the speed would increase and the orbit time would decrease, because $v \propto r^{-1 / 2}$ whilst $T \propto r^{3 / 2}$.

Direct application of Newton's law of gravitation easily gave the answer in this question, which had a facility of $78 \%$. A very small number of incorrect responses came from assuming that the law gives $F \propto(1 / r)$ - represented by distractors A and D. Rather more (14\%) chose distractor B; these students probably added the two component forces acting on the spacecraft instead of subtracting them.

This question tested students' knowledge of gravitational potential. $61 \%$ of the answers were correct. Each incorrect answer attracted a significant proportion of the responses, the most common being distractor $\mathrm{D}(19 \%)$. This choice came from confusing the correct unit of gravitational potential, $\mathrm{Jkg}^{-1}$, with the unit of field strength $\left(\mathrm{N} \mathrm{kg}^{-1}\right)$.

In this question the candidates had to decide about the relative magnitudes of the forces from the Earth, the Moon and the Sun acting on a spacecraft when close to the Earth. Values for the relevant masses and distances were provided in case candidates needed to perform a calculation, or to carry out a check on their intuition. Obviously the spacecraft would not be in orbit around Earth if $F_{E}$ was smaller than either of the other two forces. Hence $F_{E}$ must be the largest of the three forces. The relative sizes of $F_{M}$ and $F_{S}$ then comes down to the ratio $M / R^{2}$, because the local gravitational field strength caused by each of the masses is $G M / R^{2}$. The facility of the question was $56 \%$. $29 \%$ of the candidates chose distractor C ; they appreciated that $F_{E}$ is largest but thought that $F_{M}$ would be greater than $F_{S}$.

In this question there were two further tests of gravitational field strength which candidates found demanding, with facilities of $46 \%$ and $50 \%$ respectively. The question required candidates to find the ratio $R_{\mathrm{E}}: R_{\mathrm{M}}$ when $g_{\mathrm{E}}: g_{\mathrm{M}}$ are in the ratio 9.8:1.7 and $M_{\mathrm{E}}$ is $81 \times M_{\mathrm{M}}$. Forgetting to take the square root of $\left(R_{\mathrm{E}} / R_{\mathrm{M}}\right)^{2} 2$ when applying the equation $g=G M / R^{2}$ was probably responsible for the incorrect response of the $27 \%$ of the candidates who chose distractor C .

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